Comment on "Small-scale intermittency in randomly stirred fluids"

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We feel that the paper under consideration [M. Nandy, Phys. Rev. E **48**, 1015 (1993)] contains some inaccuracies regarding the dimensional analysis used in the discussion of the corrections to Kolmogorov scaling laws. Our remarks are as follows. (i) Incorrect diagram equations have been used under the dimensional self-consistency analysis. (ii) From the hydrodynamic Ward identity it follows that a scale dimension of the vertex is canonical. (iii) Simple dimensionality arguments are invalid when integrals in diagram equations contain divergences, as it is for Kolmogorov's case. (iv) A multiplicative velocity renormalization is inconsistent with Dyson diagram equations.

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In Ref. [1] the author considers an effect of the vertex renormalization on scaling properties of the turbulent field in addition to the viscosity renormalization usually used in turbulence theory. As a result a new parameter determining a scale dimension of the vertex appears. Nandy proposes to search for a relation between the scale dimensions of a frequency and the vertex on the basis of a dimensional self-consistency of the diagram equations for the self-energy operator and the vertex. Our remarks are as follows.

(i) The scale analysis in the paper is based on incorrect diagram equations. Following Wyld [2] Nandy substitutes renormalized vertices for all bare ones in diagram equations. As was first pointed out by Lee [3] some of the vertices need to remain unrenormalized in order to avoid double counting for some diagrams. Correct diagram equations may be obtained beyond the scope of the perturbation theory and they have the form [4] (in the notations used by Nandy) as shown in Figs. 1 and 2. In Fig. 1 and Fig. 2 vertices of the second and third types following from a direct analysis of the perturbation diagram series [5] are omitted.

(ii) A connection between the scale dimensions of the self-energy operator and the vertex follows from the Ward identity which is a rigorous consequence of Galilean invariance of the hydrodynamic equations [6]. According to this identity for a vertex Γ, containing one external line with a wave number and frequency equal to zero, one has

$$\Gamma_{jlm}(\mathbf{k},\omega|\mathbf{k},\omega;0,0) = -k_m \frac{\partial}{\partial\omega} G_{jl}^{-1}(\mathbf{k},\omega) = P_{jlm}(\mathbf{k})\lambda(k,\omega/k^z).$$

From this relation it follows that the scale dimension of the vertex is canonical, that is, it remains unchanged under renormalization in contrast to Nandy's assumption.

(iii) A determination of scale dimensions based on a requirement of self-consistency of the diagram equations is valid only if divergences of appropriate integrals are absent. But in Kolmogorov's case $y = 4$ the integrals in equations Fig. 1 and Fig. 2 contain divergences in the infrared domain. A separation of the sweeping effects excludes the strongest divergences and after that only logarithmic divergences will remain. A summing over these divergences within the framework of the renormalization-group (RNG) method leads to the appearance of anomalous dimensions which corresponds to accounting for intermittency effects [7].

(iv) A multiplicative renormalization of the velocity field is inconsistent with Dyson's equations for the velocity correlation function and Green's function. As has been shown in [6] the only possible renormalization is a renormalization of the external stirring force amplitude.

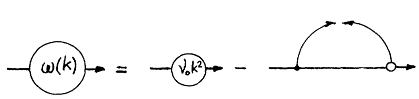

FIG. 1. Renormalization of viscosity.

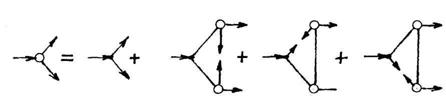

FIG. 2. Vertex renormalization.

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